Question 1 (25 minutes)

The graph of \( g(x) \) for \(-6 \leq x \leq 8\) is shown above, which for \(-2 \leq x \leq 0\) is part of the circle centered at \((-2, 0)\). Let \( f(x) = \int_{0}^{x} g(t) \, dt \).

(a) Find the average rate of change of \( g(x) \) on \([-6, 8]\).

(b) \( f(0) = 0 \). Does there exist another value of \( x \) on \(-6 \leq x \leq 8\) such that \( f(x) = 0 \)? Justify your response.

(c) Determine all intervals on which \( f(x) \) is concave down.

(d) Find the maximum value of \( f(x) \) on \([-6, 8]\). Justify your response.

(e) Write the second-degree Taylor polynomial for \( f(x) \) centered at \( x = 4 \).
Question 2 (15 minutes)

The functions \( f(x) \) and \( f'(x) \) are defined on \( 1 \leq x \leq 4 \) and \( |f'(x)| \leq 4 \) for \( 1 \leq x \leq 4 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>6.2</td>
<td>5.0</td>
<td>4.6</td>
<td>4.8</td>
<td>5.2</td>
<td>5.8</td>
<td>6.8</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>-2.0</td>
<td>-1.2</td>
<td>0.5</td>
<td>0.8</td>
<td>1.2</td>
<td>2.4</td>
<td>1.8</td>
</tr>
</tbody>
</table>

(a) Estimate the value of \( f'(2.5) \).

(b) Use a left-endpoint Riemann sum with three equal subinterval to estimate the average value of \( f(x) \) on \( 1 \leq x \leq 4 \).

(c) Explain why \( f(2.25) < 6 \).

(d) The region bounded by the function \( y = f(x) \) and the \( x \)-axis for \( 1 \leq x \leq 4 \) is revolved about the line \( y = 10 \). Set up an integral expression involving \( f(x) \) for the volume of the resulting solid of revolution.