Question 1 (25 minutes)

For a function $f(x)$ it is known that $f(0) = 7$, $f''(x)$ is continuous, and $f''(x) < 0$ for $0 \leq x \leq 6$. Select values of the first and second derivatives of $f(x)$ are given in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>-1</td>
<td>-1.5</td>
<td>-3</td>
</tr>
<tr>
<td>$f''(x)$</td>
<td>-3</td>
<td>-2</td>
<td>-5</td>
<td>-2</td>
<td>-1</td>
<td>-3</td>
<td>-6</td>
</tr>
</tbody>
</table>

(a) Using Euler's method and two steps of equal size, estimate $f(6)$ from the value of $f(0)$.

(b) Use the table of values and a trapezoidal sum using three equal subintervals to estimate $f(6)$.

(c) Explain why $f(x)$ must take a relative maximum on the interval $(0,6)$.

(d) Let $g(x) = f(3x)$. Write the second-degree Taylor polynomial for $g(x)$ centered at $x = 0$. 


Question 2 (15 minutes)

The rate of cars entering a parking lot, measured in cars per hour, is modeled by the function 
\[ f(t) = 50 \left( \sqrt{t} + 2 \cos \left( \frac{1}{7} t^2 \right) \right), \]
where \( 0 \leq t \leq 8 \) is the time, in hours, after 12 noon (\( t = 0 \) is noon). At \( t = 0 \) the parking lot is empty.

(a) Compute \( f'(3) \) and provide the appropriate units. Interpret the meaning of the answer in the context of the problem.

(b) How many cars enter the parking lot from noon to 8 pm? Give your answer to the nearest car.

(c) How many cars are in the lot when the rate at which cars are entering is at a minimum for \( 0 \leq t \leq 8 \) ? Give your answer to nearest car.

(d) The parking lot uses a dynamic pricing system, charging \( 10 - t \) dollars to enter before 5 pm, and $5 to enter after 5 pm. To the nearest dollar, how much money is collected from \( t = 0 \) to \( t = 8 \) ?