

**Question 1 (25 minutes)**

For a function  $f(x)$  it is known that  $f(0)=7$ ,  $f''(x)$  is continuous, and  $f''(x)<0$  for  $0\leq x\leq 6$ . Select values of the first and second derivatives of  $f(x)$  are given in the table below.

$x$	0	1	2	3	4	5	6
$f'(x)$	5	4	3	1	-1	-1.5	-3
$f''(x)$	-3	-2	-5	-2	-1	-3	-6

- (a) Using Euler's method and two steps of equal size, estimate  $f(6)$  from the value of  $f(0)$ .
- (b) Use the table of values and a trapezoidal sum using three equal subintervals to estimate  $f(6)$ .
- (c) Explain why  $f(x)$  must take a relative maximum on the interval  $(0,6)$ .
- (d) Let  $g(x)=f(3x)$ . Write the second-degree Taylor polynomial for  $g(x)$  centered at  $x=0$ .

**Question 2 (15 minutes)**

The rate of cars entering a parking lot, measured in cars per hour, is modeled by the function

$f(t) = 50\left(\sqrt{t} + 2\cos\left(\frac{1}{7}t^2\right)\right)$ , where  $0 \leq t \leq 8$  is the time, in hours, after 12 noon ( $t = 0$  is noon). At  $t = 0$  the parking lot is empty.

- (a) Compute  $f'(3)$  and provide the appropriate units. Interpret the meaning of the answer in the context of the problem.
- (b) How many cars enter the parking lot from noon to 8 pm? Give your answer to the nearest car.
- (c) How many cars are in the lot when the rate at which cars are entering is at a minimum for  $0 \leq t \leq 8$ ? Give your answer to nearest car.
- (d) The parking lot uses a dynamic pricing system, charging  $10 - t$  dollars to enter before 5 pm, and \$5 to enter after 5 pm. To the nearest dollar, how much money is collected from  $t = 0$  to  $t = 8$ ?